International Journal of Networking and Computing – www.ijnc.org
ISSN 2185-2839 (print) ISSN 2185-2847 (online)
Volume 4, Number 2, pages 251–259, July 2014

Algorithmic aspects of distance constrained labeling: a survey

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Received: May 2, 2014
Revised: June 1, 2014
Accepted: June 3, 2014
Communicated by Koji Nakano

Abstract

Distance constrained labeling problems, e.g., $L(p, q)$-labeling and (p, q)-total labeling, are originally motivated by the frequency assignment. From the viewpoint of theory, the upper bounds on the labeling numbers and the time complexity of finding a minimum labeling are intensively and extensively studied. In this paper, we survey the distance constrained labeling problems from algorithmic aspects, that is, computational complexity, approximability, exact computation, and so on.

Keywords: distance constrained labeling, $L(2, 1)$-labeling, $(2, 1)$-total labeling, frequency assignment

1 Introduction

Let G be an undirected graph. An $L(p, q)$-*labeling* of a graph G is an assignment f from the vertex set $V(G)$ to the set of nonnegative integers such that $|f(x) - f(y)| \geq p$ if x and y are adjacent and

[0]A part of this work was presented in [35, 57]. This work is partially supported by KAKENHI 23500022, 24700001, 24106004, 25104521, 25106508 and 26540005, the Kayamori Foundation of Informational Science Advancement, and the Asahi Glass Foundation.

$|f(x) - f(y)| \geq q$ if x and y are at distance 2, for all x and y in $V(G)$. A k-$L(p,q)$-labeling is an $L(p,q)$-labeling $f : V(G) \to \{0,\ldots,k\}$, and the *$L(p,q)$-labeling problem* asks the minimum k among all possible assignments. We call this invariant, the minimum value k, the *$L(p,q)$-labeling number* and is denoted by $\lambda_{p,q}(G)$. Notice that we can use $k+1$ different labels when $\lambda_{p,q}(G) = k$ since we can use 0 as a label for conventional reasons. Instead of $L(p,q)$-labeling, term "$L(p,q)$-coloring" is sometimes used (e.g., [14]). Here, p and q are parameters, and in a general setting of $L(p,q)$-labeling problem, they are defined in advance. Among many candidates of pairs p and q, $p = 2$ and $q = 1$ is the most popular setting, but other settings such as $p = q = 1$ and $p = 0, q = 1$ are also studied from a viewpoint of practical applications (e.g., [4, 7]).

A *(p,q)-total labeling* of a graph G is an assignment f from the vertex set $V(G)$ and the edge set $E(G)$ to the set of nonnegative integers such that $|f(x) - f(y)| \geq p$ if x is a vertex and y is an edge incident to x, and $|f(x) - f(y)| \geq q$ if x and y are a pair of adjacent vertices or a pair of adjacent edges, for all x and y in $V(G) \cup E(G)$. A k-(p,q)-total labeling is a (p,q)-total labeling $f : V(G) \cup E(G) \to \{0,\ldots,k\}$, and the *$(p,q)$-total labeling problem* asks the minimum k among all possible assignments. We call this invariant, the minimum value k, the *(p,q)-total labeling number* and is denoted by $\lambda^T_{p,q}(G)$. $L(p,q)$-labeling and (p,q)-total labeling are examples of *distance constrained labelings*, which are intensively and extensively studied from the theoretical and applicational points of view. The notion of (p,q)-total labeling is introduced by Havet and Yu [42], but it is a special case of $L(p,q)$-labeling, because a (p,q)-total labeling of G corresponds to an $L(p,q)$-labeling of the incidence graph of G, where the incidence graph of G is the graph obtained from G by replacing each edge (v_i, v_j) with two edges (v_i, v_{ij}) and (v_{ij}, v_j) after introducing one new vertex v_{ij}.

The original notion of $L(p,q)$-labeling or $L(2,1)$-labeling can be seen in Hale [32] and Roberts [59] in the context of frequency assignment, where 'close' transmitters must receive different frequencies and 'very close' transmitters must receive frequencies that are at least two frequencies apart so that they can avoid interference. Then, Griggs and Yeh formally introduced the notion of $L(p,q)$-labeling in [31, 67]. Due to its practical importance, the $L(2,1)$-labeling problem has been widely studied. From the structural graph theoretical point of view, since this is a kind of vertex coloring problem, it has attracted a lot of interest [16, 31, 40, 65]. Especially, deriving upper bounds on $\lambda_{2,1}$ (or more generally $\lambda_{p,q}$) for general graphs is one of the main concern. Griggs and Yeh [31] posed a conjecture that $\lambda_{2,1} \leq \Delta^2$ for any graph with $\Delta \geq 2$, where Δ is the maximum degree of G, and they proved that $\lambda_{2,1} \leq \Delta^2 + 2\Delta$ at the same time. After that, it was shown that $\lambda_{2,1} \leq \Delta^2 + \Delta$ by Chang and Kuo [16], $\lambda_{2,1} \leq \Delta^2 + \Delta - 1$ by Král' and Škrekovski [51], and then $\lambda_{2,1} \leq \Delta^2 + \Delta - 2$ by Gonçalves [30], however, the conjecture is still open. In this context, $L(2,1)$-labeling is generalized into $L(p,q)$-labeling for arbitrary nonnegative integers p and q, and in fact, we can see that $L(1,0)$-labeling ($L(p,0)$-labeling, actually) is equivalent to the classical vertex coloring. We can find a lot of related results on $L(p,q)$-labelings in comprehensive surveys by Calamoneri [11, 12] and by Yeh [68]. The survey paper [12] is still updated and we can download the latest version from a web page [1]. The current latest version is ver. March 14, 2011.

In the following sections, we first see the computational complexity of $L(p,q)$-labeling problems, and then see the bounds on (p,q)-total labeling numbers.

2 The computational complexity of $L(p,q)$-labeling problems

There are also a number of studies on the $L(p,q)$-labeling problem from the algorithmic point of view. It should be noted that for a graph G and a positive integer c, G has a ck-$L(cp,cq)$-labeling if and only if G has a k-$L(p,q)$-labeling (Observation 5.2 in [27]). That is, $L(cp,cq)$-labeling problem and $L(p,q)$-labeling problem are equivalent in the sense of polynomial-time computability; we assume that p and q has no common divisor hereafter.

Unfortunately, for any positive integers p and q with $p \geq q$, $L(p,q)$-labeling problem is NP-hard. More precisely, for any p and q, there exists a k such that it is NP-complete to decide whether G has a k-$L(p,q)$-labeling. If $q = 0$, it is obviously NP-hard, due to the NP-hardness of the ordinary

[1] `http://www.dsi.uniroma1.it/~calamo/PDF-FILES/survey.pdf`

coloring problem. For another basic case of $p = q$ (i.e., it is equivalent to $L(1,1)$-labeling problem), it is also shown to be NP-hard in [54], where the problem is called Distance-2 Graph Coloring Problem. The $L(1,1)$-labeling problem is well studied also from the viewpoint of restricted classes of graphs, such as planar graphs [2, 10, 55, 60, 66], outerplanar graphs [3], and so on [8, 53].

For the more general cases $p > q \geq 1$, Fiala, Kloks and Kratochvíl showed that it is NP-complete to decide whether G has a $(p+q\lceil p/q\rceil)$-$L(p,q)$-labeling, and conjectured that there exists some value $K_{p,q}$ such that for every integer $k \geq K_{p,q}$ it is NP-hard to decide G has a k-$L(p,q)$-labeling [27]. They partially succeeded to show it; it is true when $p > 2q$. Also, for $p > 2$, the problem of deciding whether G has k-$L(p,1)$-labeling is NP-hard for every $k \geq p+5$, and is solvable in polynomial time for $k \leq p+2$ [27]. For the case of $p < q$, only a few results are known: For example, the $L(0,1)$-labeling problem is NP-hard [7], and bounds on $L(p,q)$-labeling numbers of trees with $p < q$ are investigated in [13].

Due to the NP-hardness, it is natural to investigate its approximability. For $L(1,1)$-labeling problem, it is shown that a greedy algorithm achieves $O(\sqrt{n})$-approximation ratio [1, 54], and it is hard to approximate in polynomial time within factor $n^{1/2-\varepsilon}$ for any $\varepsilon > 0$, unless NP$\neq$ZPP [1]. This holds even if the input graph is bipartite or a split graph. For general p and q, the following result is known: it can be approximate in polynomial time within a factor of $O(\min\{\Delta, \sqrt{n}+p/q\})$, but it is NP-hard to approximate within factor $n^{1/2-\varepsilon}$ for any $\varepsilon > 0$ [33].

2.1 $L(2,1)$-labeling problem

As mentioned above, the most popular parameter setting of $L(p,q)$-labeling is $p = 2, q = 1$. First, it was shown to be NP-hard to decide whether a given graph G has a k-$L(2,1)$-labeling for some integer k [31]. Actually, this holds for every integer $k \geq 4$ [27]. It was then shown that it still remains NP-hard for some restricted classes of graphs, such as planar graphs, bipartite graphs, chordal graphs [9] and graphs with diameter 2 [31]. Particularly, in the case of planar graphs, determining the existence of k-$L(2,1)$-labeling is NP-hard even for every $k \geq 4$, while it can be done in polynomial time for $k \leq 3$ [20]. Also, it is NP-hard even for graphs of treewidth 2 [22] and for perfect elimination bipartite graphs [58]. In contrast, only a few graph classes are known to have polynomial-time algorithms for this problem, e.g., we can determine in polynomial time the $L(2,1)$-labeling number of paths, cycles, wheels [31] and co-graphs [16].

As for trees, Griggs and Yeh [31] showed that $\lambda_{2,1}(T)$ is either $\Delta+1$ or $\Delta+2$ for any tree T, and also conjectured that determining $\lambda_{2,1}(T)$ is NP-hard, however, Chang and Kuo [16] disproved this by presenting a polynomial-time algorithm for computing $\lambda_{2,1}(T)$. Their algorithm exploits the fact that $\lambda_{2,1}(T)$ is either $\Delta+1$ or $\Delta+2$ for any tree T. Its running time is $O(\Delta^{4.5}n)$, where $n = |V(T)|$. This result has a great importance because it initiates to cultivate polynomially solvable classes of graphs for the $L(2,1)$-labeling problem and related problems. For example, Fiala, Kloks and Kratochvíl [27] showed that $L(2,1)$-labeling of t-almost trees can be solved in $O(\lambda_{2,1}^{2t+4.5}n)$ time for $\lambda_{2,1}$ given as an input, where a t-almost tree is a graph that can be a tree by eliminating t edges. Also, it was shown that the $L(p,1)$-labeling problem for trees can be solved in $O((p+\Delta)^{5.5}n) = O(\lambda_{2,1}^{5.5}n)$ time [15]. Both results are based on Chang and Kuo's algorithm, which is called as a subroutine in the algorithms. Moreover, the polynomially solvable result for trees holds for more general settings. The notion of $L(p,1)$-labeling is generalized as $H(p,1)$-labeling, in which graph H defines the metric space of distances between two labels, whereas labels in $L(p,1)$-labeling (that is, in $L(p,q)$-labeling) take nonnegative integers; i.e., it is a special case that H is a path graph. In [24], it has been shown that the $H(p,1)$-labeling problem of trees for arbitrary graph H can be solved in polynomial time, which is also based on Chang and Kuo's idea. In passing, these results are unfortunately not applicable for $L(p,q)$-labeling problems for general p and q. Fiala, Golovach and Kratochvíl [23] showed that the $L(p,q)$-labeling problem for trees is NP-hard if q is not a divisor of p, which is contrasting to the positive results mentioned above.

As for $L(2,1)$-labeling of trees again, Chang and Kuo's $O(\Delta^{4.5}n)$ algorithm is the first polynomial-time one. It is based on dynamic programming (DP) approach, and it checks whether $(\Delta+1)$-$L(2,1)$-labeling is possible or not from leaf vertices to a root vertex in the original tree structure. The principle of optimality requires to solve at each vertex of the tree the assignments of labels to subtrees, and the assignments are formulated as the maximum matching in a certain bipartite graph. This running time is improved into $O(\min\{n^{1.75}, \Delta^{1.5}n\})$ [34], and recently, a linear time algorithm has been established [38]. They are based on the similar DP framework to Chang and Kuo's algorithm, but achieve their efficiency by reducing heavy computation of bipartite matching in Chang and Kuo's and by using an amortized analysis. Particularly, the latter algorithm achieves the linear running time by best utilizing a nice property of labeling, called *label compatibility*. Since this property holds for more general labelings, say $L(p,1)$-labeling, the linear time algorithm for $L(2,1)$-labeling of trees can be extended to a linear time algorithm for $L(p,1)$-labeling of trees for a fixed positive integer p.

As an intermediate class between graphs with treewidth 2 and trees (i.e., graphs with treewidth 1), outerplanar graphs are known. For outerplanar graphs, a polynomial-time algorithm is known, though the degree of the polynomial is large [48]. As far as the authors know, outerplanar graphs are the most well-known maximal graph class that has a polynomial-time algorithm solving the $L(2,1)$-labeling problem.

As mentioned at the beginning of this section, it is NP-hard to find an optimal $L(2,1)$-labeling of a given graph, and we cannot expect a polynomial-time algorithm. Then, designing a faster exponential-time algorithm becomes meaningful. As a research to this direction, Král' first presented an exact algorithm for a more general labeling problem, called *channel assignment problem*. The algorithm solves $L(p,q)$-labeling problem in $O(n(\max\{p,q\}+2)^n)$ time and space $O((\max\{p,q\}+2))$, so it solves $L(2,1)$-labeling problem in $O^*(4^n)$, where polynomial terms are omitted in O^* notation [50]. In [39], Havet et al. propose an $O^*(3.8730^n)$-time algorithm for $L(2,1)$-labeling problem, and the time complexity is improved by Junosza-Szaniawski et al. to $O^*(3.2361^n)$ [46]. Then, Cygan and Kowalik presented a faster algorithm for the channel assignment problem, based on the fast zeta transform in combination with the inclusion-exclusion principle [18]. The algorithm solves $L(p,q)$-labeling problem in $O^*((\max\{p,q\}+1)^n)$ time and $L(2,1)$-labeling problem in $O^*(3^n)$ time. Junosza-Szaniawski et al. [44] then present a faster algorithm. Its running time is $O^*(2.6488^n)$ and currently fastest. Note that all of these algorithms use exponential size of memories. The current fastest exact algorithm with polynomial space for $L(2,1)$-labeling is proposed by Junosza-Szaniawski et al. [45], and it runs in $O^*(7.4922^n)$ time.

Another issue about the exact computation is parameterized complexity. For a given parameter α, a problem is called *fixed parameter tractable* with respect to α if it can be solved in $f(\alpha)\cdot n^{O(1)}$ time, where f is some computable function. As seen above, since $L(2,1)$-labeling problem is NP-hard even for graphs with treewidth 2 or even for $k=4$, it is not fixed parameter tractable with respect to treewidth or $\lambda_{2,1}$, unless P=NP. In contrast, it is fixed parameter tractable with respect to the vertex cover number [25].

3 (p,q)-total labeling

We review several results on (p,q)-total labeling.

We notice that $(1,1)$-total labeling of G is equivalent to total coloring of G. Generalizing the Total Coloring Conjecture [6, 63], Havet and Yu [42] conjectured that

$$\lambda^T_{p,1}(G) \le \Delta + 2p - 1$$

holds for any graph G. They also investigated bounds on $\lambda^T_{p,1}(G)$ under various assumptions and some of their results are described as follows: (i) $\lambda^T_{p,1}(G) \ge \Delta+p-1$, (ii) $\lambda^T_{p,1}(G) \ge \Delta+p$ if $p \ge \Delta$, (iii) $\lambda^T_{p,1}(G) \le \min\{2\Delta+p-1, \chi(G)+\chi'(G)+p-2\}$ for any graph G, where $\chi(G)$ and $\chi'(G)$ denote the chromatic number and the chromatic index of G, respectively, and (iv) $\lambda^T_{p,1}(G) \le n+2p-2$ if G is the complete graph where $n=|V(G)|$. In particular, it follows by (iii) that if G is bipartite, then $\lambda^T_{p,1}(G) \le \Delta+p$ holds (by $\chi(G)\le 2$ and $\chi'(G)=\Delta$ [49]), and if in addition, $p\ge\Delta$, then $\lambda^T_{p,1}(G)=\Delta+p$ by (ii) [5, 42]. Also, Bazzaro et al. [5] showed that $\lambda^T_{p,1}(G)\le\Delta+p+s$ for any s-degenerated graph (by $\chi(G)\le s+1$ and $\chi'(G)\le\Delta+1$ [62]), where an *s-degenerated graph* G is a graph which can be reduced to a trivial graph by successive removal of vertices with degree at most s, that

$\lambda^T_{p,1}(G) \le \Delta+p+3$ for any planar graph (by the Four-Color Theorem), and that $\lambda^T_{p,1}(G) \le \Delta+p+1$ for any outerplanar graph other than an odd cycle (since any outerplanar graph is 2-degenerated, and any outerplanar graph other than an odd cycle satisfies $\chi'(G) = \Delta$ [28]). As for the $(2,1)$-total labeling number of outerplanar graphs is known to be at most $\Delta+2$, which is tight, i.e., there exists an outerplanar graph whose $(2,1)$-total labeling number is $\Delta + 2$ [36, 37]. Also, there are many related works about bounds on $\lambda^T_{p,1}(G)$ [17, 38, 52, 56]. From the algorithmic point of view, Havet and Thomassé [41] showed that for bipartite graphs, if (i) $p \ge \Delta$ or (ii) $\Delta = 3$ and $p = 2$, then the $(p,1)$-total labeling problem is polynomially solvable and otherwise it is NP-hard.

In [19, 47, 61], the $[r,s,t]$-coloring problem which is a generalization of the (p,q)-total labeling problem was studied, while results in the cases corresponding to the (p,q)-total labeling problem (actually, the cases of $t \ge r = s$) are limited to paths, cycles, stars or the complete graph with some p and q.

As for the (p,q)-total labeling of trees, the following results are known [37]:

- (Upper bounds on $\lambda^T_{p,q}(T)$) If $p = q + r$ for $r \in \{0,1,\dots,q-1\}$ and $\Delta > 1$ (resp., $\Delta = 1$), then $\lambda^T_{p,q}(T) \le p + (\Delta-1)q + r$ holds and this bound is tight (resp., $\lambda^T_{p,q}(T) = p + q$). If $p \ge 2q$, then $\lambda^T_{p,q}(T) \le p + \Delta q$ holds and this bound is tight. In particular, if $p \ge \Delta q$, then $\lambda^T_{p,q}(T) = p + \Delta q$.

- (Lower bounds on $\lambda^T_{p,q}(T)$) If $q \le p < (\Delta-1)q$, then $\lambda^T_{p,q}(T) \ge p + (\Delta-1)q$ holds and this bound is tight. If $p = (\Delta-1)q + r$ for $r \in \{0,1,\dots,q-1\}$, then $\lambda^T_{p,q}(T) \ge p + (\Delta-1)q + r$ holds and this bound is tight. If $p \ge \Delta q$, then $\lambda^T_{p,q}(T) = p + \Delta q$.

- The (p,q)-total labeling problem with $p \le 3q/2$ for trees can be solved in linear time. In particular, if $\Delta \ge 2$, we have $\lambda^T_{p,q}(T) \in \{p + (\Delta-1)q, p + (\Delta-1)q + r\}$. If $p > q$ and $\Delta \ge 4$, then $\lambda^T_{p,q}(T) = p + (\Delta-1)q$ holds if and only if no two vertices with degree Δ are adjacent.

- In the case of $p = 2q$, the condition that no two vertices with degree Δ are adjacent is sufficient for $\lambda^T_{p,q}(T) = p + (\Delta-1)q$, while in the case of $p > 3q/2$ and $p \ne 2q$, this condition is not sufficient.

- For any two nonnegative integers p and q, the $L(p,q)$-labeling problem for trees can be solved in polynomial time if $\Delta = \mathrm{O}(\log^{1/3}|I|)$ where $|I| = \max\{|V(T)|, \log p\}$. Particularly, if Δ is a fixed constant, it is solved in linear time.

The first and second results provide tight upper and lower bounds on $\lambda^T_{p,q}(T)$ for all pairs (p,q) with $p \ge q$. The first statement in the third result indicates that as for the (p,q)-total labeling problem for trees, there exists a tractable case even if q is not a divisor of p, in contrast to the NP-hardness of the $L(p,q)$-labeling problem. The second and third statements in the third result completely characterize trees T achieving $\lambda^T_{p,q}(T)$ in the case of $p \le 3q/2$ and $\Delta \ge 4$ (note that if $p = q$, we have $\lambda^T_{p,q}(T) = p + (\Delta-1)q$ by the first and second results). This is also contrasting to the fact that no simple characterization of trees T achieving $\lambda_{2,1}(T)$ is known even for the $L(2,1)$-labeling problem.

4 Other results

Other than $L(p,q)$-labeling and (p,q)-total labeling, many distance constrained labeling problems are considered. A natural generalization of $L(p,q)$-labeling is $L(p_1,p_2,\dots,p_\ell)$-labeling. In fact, several publications mentioned above actually study $L(p_1,p_2,\dots,p_\ell)$-labeling (e.g., [2, 54]), although the main results are about $L(p_1,p_2)$-labeling (or even $L(2,1)$-labeling) in many cases.

The following introduces some miscellaneous results. Fiala et al. [21] consider online and offline setting of $L(p_1,p_2,\dots,p_\ell)$-labeling of disk graphs. Also, “distance three labeling” is studied as a natural and not too generalized extension. Unfortunately, $L(2,1,1)$-labeling problem of trees is already NP-hard [26].

From the viewpoint of channel assignments, the list version of $L(p,q)$-labeling could be meaningful. The *list-coloring* is a list version of the ordinary coloring, and is actually studied in the context of channel assignments [29, 64]. It is a generalization of "coloring" for vertices (i.e., the colors assigned to adjacent vertices differ), where the color of vertex v should be chosen from its own color list $L(v)$. In channel assignments, it is not allowed to assign some channels (frequencies) to some wireless nodes due to several practical constraints, and color lists can reflect such constraints. Thus, it is also natural to consider the list version of $L(p,q)$-labeling or (p,q)-total labeling. Ito et al. [43] studies reassignments of the list version of $L(2,1)$-labeling from such a viewpoint.

References

[1] Geir Agnarsson, Raymond Greenlaw, and Magnús M Halldórsson. On powers of chordal graphs and their colorings. *Congressus Numerantium*, pages 41–66, 2000.

[2] Geir Agnarsson and Magnús M. Halldórsson. Coloring powers of planar graphs. In *Proceedings of the eleventh annual ACM-SIAM Symposium on Discrete Algorithms*, pages 654–662. Society for Industrial and Applied Mathematics, 2000.

[3] Geir Agnarsson and Magnús M. Halldórsson. On colorings of squares of outerplanar graphs. In *Proceedings of the fifteenth annual ACM-SIAM Symposium on Discrete algorithms*, pages 244–253. Society for Industrial and Applied Mathematics, 2004.

[4] Khaled A. Aly and Patrick W. Dowd. A class of scalable optical interconnection networks through discrete broadcast-select multi-domain wdm. In *INFOCOM'94. Networking for Global Communications., 13th Proceedings IEEE*, pages 392–399. IEEE, 1994.

[5] Fabrice Bazzaro, Mickaël Montassier, and André Raspaud. $(d,1)$-total labelling of planar graphs with large girth and high maximum degree. *Discrete Mathematics*, 307(16):2141–2151, 2007.

[6] Mehdi Behzad. *Graphs and their chromatic numbers.* PhD thesis, Michigan State University. Department of Mathematics, 1965.

[7] Alan A. Bertossi and Maurizio A. Bonuccelli. Code assignment for hidden terminal interference avoidance in multihop packet radio networks. *IEEE/ACM Transactions on Networking (TON)*, 3(4):441–449, 1995.

[8] Norman Biggs. Colouring square lattice graphs. *Bulletin of the London Mathematical Society*, 9(1):54–56, 1977.

[9] Hans L. Bodlaender, Ton Kloks, Richard B. Tan, and Jan Van Leeuwen. Approximations for λ-colorings of graphs. *The Computer Journal*, 47(2):193–204, 2004.

[10] O.V. Borodin, H.J. Broersma, A. Glebov, and van den J. Heuvel. Stars and bunches in planar graphs. part II: General planar graphs and colourings. Technical report, Department of Applied Mathematics, University of Twente, 2002.

[11] Tiziana Calamoneri. The $L(h,k)$-labelling problem: A survey and annotated bibliography. *The Computer Journal*, 49(5):585–608, 2006.

[12] Tiziana Calamoneri. The $L(h,k)$-labelling problem: an updated survey and annotated bibliography. *The Computer Journal*, 54(8):1344–1371, 2011.

[13] Tiziana Calamoneri, Andrzej Pelc, and Rossella Petreschi. Labeling trees with a condition at distance two. *Discrete Mathematics*, 306(14):1534–1539, 2006.

[14] Márcia R. Cerioli and Daniel F.D. Posner. On $L(2,1)$-coloring split, chordal bipartite, and weakly chordal graphs. *Discrete Applied Mathematics*, 160(18):2655–2661, 2012.

[15] Gerard J. Chang, Wen-Tsai Ke, David Kuo, Daphne D-F. Liu, and Roger K. Yeh. On $L(d,1)$-labelings of graphs. *Discrete Mathematics*, 220(1):57–66, 2000.

[16] Gerard J. Chang and David Kuo. The $L(2,1)$-labeling problem on graphs. *SIAM Journal on Discrete Mathematics*, 9(2):309–316, 1996.

[17] Dong Chen and Weifan Wang. $(2,1)$-total labelling of outerplanar graphs. *Discrete Applied Mathematics*, 155(18):2585–2593, 2007.

[18] Marek Cygan and Łukasz Kowalik. Channel assignment via fast zeta transform. *Information Processing Letters*, 111(15):727–730, 2011.

[19] Lyes Dekar, Brice Effantin, and Hamamache Kheddouci. $[r,s,t]$-coloring of trees and bipartite graphs. *Discrete Mathematics*, 310(2):260–269, 2010.

[20] Nicole Eggemann, Frédéric Havet, and Steven D. Noble. k-$L(2,1)$-labelling for planar graphs is np-complete for $k \geq 4$. *Discrete Applied Mathematics*, 158(16):1777–1788, 2010.

[21] Jiří Fiala, Aleksei V. Fishkin, and Fedor Fomin. On distance constrained labeling of disk graphs. *Theoretical Computer Science*, 326(1):261–292, 2004.

[22] Jiří Fiala, Petr A. Golovach, and Jan Kratochvíl. Distance constrained labelings of graphs of bounded treewidth. In *Proceedings of the 32nd International Colloquium on Automata, Languages and Programming*, pages 360–372. Springer-Verlag, 2005.

[23] Jiří Fiala, Petr A. Golovach, and Jan Kratochvíl. Computational complexity of the distance constrained labeling problem for trees. In *Proceedings of the 35th International Colloquium on Automata, Languages and Programming, Part I*, pages 294–305. Springer-Verlag, 2008.

[24] Jiří Fiala, Petr A. Golovach, and Jan Kratochvíl. Distance constrained labelings of trees. In *Proceedings of the 5th Theory and Applications of Models of Computation*, pages 125–135. Springer-Verlag, 2008.

[25] Jiří Fiala, Petr A. Golovach, and Jan Kratochvíl. Parameterized complexity of coloring problems: Treewidth versus vertex cover. *Theoretical Computer Science*, 412(23):2513–2523, 2011.

[26] Jiří Fiala, Petr A. Golovach, Jan Kratochvíl, Bernard Lidický, and Daniël Paulusma. Distance three labelings of trees. *Discrete Applied Mathematics*, 160(6):764–779, 2012.

[27] Jiří Fiala, Ton Kloks, and Jan Kratochvíl. Fixed-parameter complexity of λ-labelings. *Discrete Applied Mathematics*, 113(1):59–72, 2001.

[28] Stanley Fiorini. On the chromatic index of outerplanar graphs. *Journal of Combinatorial Theory, Series B*, 18(1):35–38, 1975.

[29] Naveen Garg, Marina Papatriantafilou, and Philippas Tsigas. Distributed list coloring: how to dynamically allocate frequencies to mobile base stations. In *Eighth IEEE Symposium on Parallel and Distributed Processing*, pages 18–25. IEEE, 1996.

[30] Daniel Gonçalves. On the $L(p,1)$-labelling of graphs. *Discrete Mathematics*, 308(8):1405–1414, 2008.

[31] Jerrold R. Griggs and Roger K. Yeh. Labelling graphs with a condition at distance 2. *SIAM Journal on Discrete Mathematics*, 5(4):586–595, 1992.

[32] William K. Hale. Frequency assignment: Theory and applications. *Proceedings of the IEEE*, 68(12):1497–1514, 1980.

[33] Magnús M. Halldórsson. Approximating the $L(h,k)$-labelling problem. *International Journal of Mobile Network Design and Innovation*, 1(2):113–117, 2006.

[34] Toru Hasunuma, Toshimasa Ishii, Hirotaka Ono, and Yushi Uno. An $O(n^{1.75})$ algorithm for $L(2,1)$-labeling of trees. *Theoretical Computer Science*, 410(38):3702–3710, 2009.

[35] Toru Hasunuma, Toshimasa Ishii, Hirotaka Ono, and Yushi Uno. Recent advances on the $L(2,1)$-labeling problem. In *The 7th Japan Conference on Computational Geometry and Graphs (JCCGG 2009)*, 2009.

[36] Toru Hasunuma, Toshimasa Ishii, Hirotaka Ono, and Yushi Uno. The $(2,1)$-total labeling number of outerplanar graphs is at most $\Delta+2$. In *Proceedings of the 21st International Conference on Combinatorial Algorithms*, pages 103–106. Springer-Verlag, 2010.

[37] Toru Hasunuma, Toshimasa Ishii, Hirotaka Ono, and Yushi Uno. The (p,q)-total labeling problem for trees. *Discrete Mathematics*, 312(8):1407–1420, 2012.

[38] Toru Hasunuma, Toshimasa Ishii, Hirotaka Ono, and Yushi Uno. A linear time algorithm for $L(2,1)$-labeling of trees. *Algorithmica*, 66(3):654–681, 2013.

[39] Frédéric Havet, Martin Klazar, Jan Kratochvíl, Dieter Kratsch, and Mathieu Liedloff. Exact algorithms for $L(2,1)$-labeling of graphs. *Algorithmica*, 59(2):169–194, 2011.

[40] Frédéric Havet, Bruce Reed, and Jean-Sébastien Sereni. $L(2,1)$-labelling of graphs. In *Proceedings of the nineteenth annual ACM-SIAM symposium on Discrete Algorithms*, pages 621–630. Society for Industrial and Applied Mathematics, 2008.

[41] Frédéric Havet and Stéphan Thomassé. Complexity of $(p,1)$-total labelling. *Discrete Applied Mathematics*, 157(13):2859–2870, 2009.

[42] Frédéric Havet and Min-Li Yu. $(p,1)$-total labelling of graphs. *Discrete Mathematics*, 308(4):496–513, 2008.

[43] Takehiro Ito, Kazuto Kawamura, Hirotaka Ono, and Xiao Zhou. Reconfiguration of list $L(2,1)$-labelings in a graph. In *Algorithms and Computation*, pages 34–43. Springer-Verlag, 2012.

[44] Konstanty Junosza-Szaniawski, Jan Kratochvíl, Mathieu Liedloff, Peter Rossmanith, and Paweł Rzążewski. Fast exact algorithm for $L(2,1)$-labeling of graphs. *Theoretical Computer Science*, 505:42–54, 2013.

[45] Konstanty Junosza-Szaniawski, Jan Kratochvíl, Mathieu Liedloff, and Paweł Rzążewski. Determining the $L(2,1)$-span in polynomial space. *Discrete Applied Mathematics*, 161(13):2052–2061, 2013.

[46] Konstanty Junosza-Szaniawski and Paweł Rzążewski. On the complexity of exact algorithm for $L(2,1)$-labeling of graphs. *Information Processing Letters*, 111(14):697–701, 2011.

[47] Arnfried Kemnitz and Massimiliano Marangio. $[r,s,t]$-colorings of graphs. *Discrete Mathematics*, 307(2):199–207, 2007.

[48] Angela Erika Koller. *The frequency assignment problem*. PhD thesis, Brunel University, School of Information Systems, Computing and Mathematics, 2005.

[49] Dénes König. Über graphen und ihre anwendung auf determinantentheorie und mengenlehre. *Mathematische Annalen*, 77(4):453–465, 1916.

[50] Daniel Král'. An exact algorithm for the channel assignment problem. *Discrete Applied Mathematics*, 145(2):326–331, 2005.

[51] Daniel Král' and Riste Škrekovski. A theorem about the channel assignment problem. *SIAM Journal on Discrete Mathematics*, 16(3):426–437, 2003.

[52] Ko-Wei Lih, Daphne Der-Fen Liu, and Weifan Wang. On $(d,1)$-total numbers of graphs. *Discrete Mathematics*, 309(12):3767–3773, 2009.

[53] Ko-Wei Lih, Wei-Fan Wang, and Xuding Zhu. Coloring the square of a k_4-minor free graph. *Discrete Mathematics*, 269(1):303–309, 2003.

[54] S. Thomas McCormick. Optimal approximation of sparse hessians and its equivalence to a graph coloring problem. *Mathematical Programming*, 26(2):153–171, 1983.

[55] Michael Molloy and Mohammad R. Salavatipour. A bound on the chromatic number of the square of a planar graph. *Journal of Combinatorial Theory, Series B*, 94(2):189–213, 2005.

[56] Mickaël Montassier and André Raspaud. $(d,1)$-total labeling of graphs with a given maximum average degree. *Journal of Graph Theory*, 51(2):93–109, 2006.

[57] Hirotaka Ono. Recent advances on distance constrained labeling problems. In *2013 First International Symposium on Computing and Networking (CANDAR)*, pages 26–29. IEEE, 2013.

[58] B.S. Panda and Preeti Goel. $L(2,1)$-labeling of perfect elimination bipartite graphs. *Discrete Applied Mathematics*, 159(16):1878–1888, 2011.

[59] Fred S. Roberts. T-colorings of graphs: recent results and open problems. *Discrete Mathematics*, 93(2):229–245, 1991.

[60] Jan van den Heuvel and Sean McGuinness. Coloring the square of a planar graph. *Journal of Graph Theory*, 42(2):110–124, 2003.

[61] Marta Salvador Villà. $[r,s,t]$*-colouring of Paths, Cycles and Stars.* PhD thesis, TU Bergakademie, Freiberg, 2005.

[62] Vadim G. Vizing. On an estimate of the chromatic class of a p-graph. *Diskret. Analiz*, 3(7):25–30, 1964.

[63] Vadim G. Vizing. Some unsolved problems in graph theory. *Russian Mathematical Surveys*, 23(6):125–141, 1968.

[64] Wei Wang and Xin Liu. List-coloring based channel allocation for open-spectrum wireless networks. In *Vehicular Technology Conference, 2005. VTC-2005-Fall. 2005 IEEE 62nd*, volume 1, pages 690–694, Sept 2005.

[65] Wei-Fan Wang. The $L(2,1)$-labelling of trees. *Discrete Applied Mathematics*, 154(3):598–603, 2006.

[66] G. Wegner. Graphs with given diameter and a coloring problem. Technical report, Technical Report, University of Dortmund, 1977.

[67] Kwan-Ching Yeh. *Labeling graphs with a condition at distance two.* PhD thesis, University of South Carolina, 1990.

[68] Roger K. Yeh. A survey on labeling graphs with a condition at distance two. *Discrete Mathematics*, 306(12):1217–1231, 2006.