# Novel List Scheduling Strategies for Data Parallelism Task Graphs 

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#### Abstract

This paper studies task scheduling algorithms which schedule a set of tasks on multiple cores so that the total scheduling length is minimized. Most of the algorithms developed in the past assume that a task is executed on a single core. Unlike the previous algorithms, the algorithms studied in this paper allow a task to be executed on multiple cores. This paper proposes six algorithms. All of the six algorithms are based on list scheduling, but the strategy for priority assignment is different. In our experiments, the six algorithms as well as an integer linear programming method are evaluated.


Keywords: task scheduling, multicore, data parallelism

## 1 Introduction

Due to the spread deployment of multicore processors not only in high-performance computers but also in embedded systems, task scheduling has now become a more important problem than ever. In general, an application is modeled as a task graph, where nodes represent tasks (i.e., pieces of the application) and direct edges represent data- or control-flow dependency between two tasks. A task scheduling problem decides when and on which core each task is executed so as to minimize the overall schedule length while meeting constraints on flow dependency and the number of cores available. Schedule length is execution time of the application. The task scheduling problem is known to be NP-hard [1], and has been extensively studied over decades to develop efficient heuristic algorithms.

Most of the previous researches assume that a task does not have data parallelism and runs on a single core, where data parallelism denotes the parallel execution of a single task on data distributed over multiple cores. However, this assumption does not hold true in many systems. Tasks may have data parallelism and run on multiple cores. This paper studies scheduling of data-parallel tasks on multicore processors.

There exist several research efforts on task scheduling with data parallelism in the past. Recent studies include [2, 3, 4]. In [2], Yang and Ha proposed a scheduling technique for data-parallel tasks based on integer linear programming (ILP) formulation, and extended the technique towards


Figure 1: An example of a task graph.
pipelined scheduling in [3]. Their techniques perform task scheduling and allocation simultaneously, where allocation means a design process which decides the number of cores assigned to each task. Vydyanathan also proposed a simultaneous scheduling and allocation algorithm for data-parallel tasks [4]. The common assumption in [2, 3] and [4] is that the degree of data parallelism in tasks, i.e., the number of cores assigned to the task, is flexible, and the execution time of the task for each parallelism is known prior to task scheduling decision. However, this assumption may not be practical in some cases.

In contrast, this paper assumes that a task has a fixed degree of data parallelism. Tasks may have different degrees of data parallelism, but the degrees are not changed during task scheduling. To the best of our knowledge, this is the first paper to propose efficient algorithms for the scheduling problem.

The contributions of this paper are as follows:

- This paper first defines and formulates the scheduling problem for a set of data-parallel tasks.
- This paper proposes six algorithms for the scheduling problem.
- This paper presents quantitative evaluations of the algorithms using standard task sets.

The rest of this paper is organized as follows. Section 2 defines the scheduling problem, and Section 3 proposes six algorithms for the problem. Experiments are shown in Section 4, and Section 5 concludes this paper.

## 2 Problem Definition

This section defines the task scheduling problem addressed in this paper.

### 2.1 Problem Description

This work assumes a homogeneous multicore processor. An application is modeled as an acyclic directed graph (DAG), so called a task graph, where a node represents a task and a directed edge represents a flow dependency between the two tasks.

Figure 1 shows an example of a task graph. In this graph, tasks labeled $S$ and $E$ are dummy tasks which do not perform any meaningful computation. Tasks $S$ and $E$ denote an entry point and an exit point of the application, respectively. Two integer values are associated with each task. The first number denotes the degree of data parallelism of the task, and the latter number denotes the execution time of the task. For example, task 1 runs on four cores, and it takes 40 time units to perform the task.

In this paper, we assume that individual tasks are written in a parallel programming language by human programmers, and that the programmers decide the degree of data parallelism. How to decide the degree of parallelism and how to know the execution time are up to the programmers, and are out of the scope of this paper.

### 2.2 ILP Formulation

The task scheduling problem described above can be formulated by an integer linear programming (ILP) problem.

Let time $_{i}$, start ${ }_{i}$, and finish $_{i}$ denote the execution time, start time and finish time of task $i$, respectively. par $r_{i}$ denotes the data parallelism, meaning that task $i$ must be mapped onto par $_{i}$ cores. flow $_{i 1, i 2}$ denotes a flow dependency between tasks $i 1$ and $i 2$. flow $w_{i 1, i 2}$ is 1 if task $i 1$ must proceed task $i 2$. map $_{i, j}$ denotes mapping of tasks on cores. $m a p_{i, j}$ is 1 if task $i$ is mapped to core $j$.

Then, the task scheduling problem is formally defined as follows: Given time ${ }_{i}$, par $_{i}$ and flow $_{i 1, i 2}$, decide start ${ }_{i}$, finish $_{i}$ and map $_{i, j}$ which minimize the objective function (1), while meeting the constraints (2), (3), (4) and (5).

Minimize:

$$
\begin{equation*}
\max _{i}\left(\text { finish }_{i}\right) \tag{1}
\end{equation*}
$$

Subject to:

$$
\begin{gather*}
\forall i \quad \sum_{j} \text { map }_{i, j}=\text { par }_{i}  \tag{2}\\
\forall i \quad \text { finish }_{i}=\text { start }_{i}+\text { time }_{i}  \tag{3}\\
\forall i 1, i 2, j \quad \text { map }_{i 1, j}+\text { map }_{i 2, j} \leq 1 \quad \text { OR } \quad \text { finish }_{i 1} \leq \text { start }_{i 2} \quad \text { OR } \quad \text { finish }_{i 2} \leq \text { start }_{i 1}  \tag{4}\\
\forall i 1, i 2 \quad \text { flow }_{i 1, i 2}=1 \rightarrow \text { finish }_{i 1} \leq \text { start }_{i 2} \tag{5}
\end{gather*}
$$

It should be noted that finish $_{i}$ is a dependent variable on $s t a r t_{i}$ (see Equation 3). Therefore, the decision variables of the scheduling problem are start $i_{i}$ and $m a p_{i, j}$. We call values of $s t a r t_{i}$ and $m a p_{i, j}$ for all $i$ and $j$ a schedule (or a scheduling result) of the task graph. A schedule is called feasible if the schedule satisfies all of the constraints (2), (3), (4) and (5). The maximum value of finish ${ }_{i}$, which is the objective function (1), is called the schedule length. Then, the scheduling problem can be restated as follows: For a given task graph, find a feasible schedule with the minimum schedule length.

Although optimal scheduling results can be obtained by solving the ILP formulas, it is not practical for large task sets in terms of CPU runtime. In the next section, we propose six heuristic algorithms based on list scheduling.

## 3 The Proposed Algorithms

In this section, we propose six algorithms for the scheduling problem. All of the six algorithms are based on list scheduling, but their priority assignment strategies are different.

### 3.1 The Overall Algorithm

The basis of the six algorithms is a simple list scheduling algorithm. An important concept of list scheduling is ReadyList, which contains a set of executable tasks. Here, a task is said to be executable if all of its preceding tasks are completed. Below is a fundamental algorithm of list scheduling.

1. Initialize ReadyList and IdleCores;

ReadyList $=\emptyset$;
IdleCores $=$ the number of total cores;
2. Select a task which has the highest priority from ReadyList, and schedule the task as early as it is schedulable;


Figure 2: Critical path length and the number of immediate successors.
3. Finish if all tasks have been scheduled. Otherwise, update ReadyList and IdleCores and go back to step 2;

There exist a large number of variations of list scheduling depending on how to define the priority in step 2.

### 3.2 A Motivating Example

In [5], Kasahara and Narita propose a list-based scheduling algorithm, named CP/MISF (critical path/most immediate successor first). The CP/MISF algorithm is designed for task scheduling without data parallelism. Although it was proposed three decades ago, it is still recognized as one of the best heuristic algorithms because of the high quality of results as well as the low computational complexity. As the name of the algorithm indicates, the CP/MISF algorithm takes into account two factors to define the priority of tasks; the critical path length and the number of immediate successors. Figure 2 shows the same task graph as in Figure 1, but we have added two numbers to each task, denoting the critical path length and the number of immediate successors. The critical path length of a task is the length of the longest path from the node to the exit node. For example, the critical path length of task 2 is 60 , by adding the execution time of task 2 and that of task 5 . In the CP/MISF algorithm, the priority of tasks is defined according to the following two rules:

1. If the critical path of task $i$ is longer than that of task $j$, task $i$ has a higher priority than task $j$.
2. In case tasks $i$ and $j$ has the same critical path length, if task $i$ has more immediate successors than task $j$, task $i$ has a higher priority than task $j$.

Figure 3 shows the schedule when the CP/MISF algorithm is applied to the task graph in Figure 2. At time $t=0$, tasks 1 and 2 are executable, but task 2 is scheduled first because it has a longer critical path. Then, tasks 5 and 4 are scheduled, followed by tasks 1 and 3 . The total schedule length is 80 time units.

The CP/MISF algorithm works nice for tasks without data parallelism. However, the CP/MISF algorithm is not always efficient for tasks with data parallelism. Actually, the schedule in Figure 3 is not optimal. Figure 4 shows a better schedule for the same task set. The policy of this scheduling is that a task with the largest data parallelism has a priority. Due to this policy, task 1 is scheduled first, and then, task 3 is enabled to run in parallel with another task. Of course, this policy is not always optimal, but this example demonstrates that the degree of data parallelism should be taken into account in the priority.

### 3.3 The Proposed Priorities

We propose six algorithms, all of which are based on list scheduling, but their definitions of priority are different. In order to define the priority, we take into account three factors as follows:

| $t=0$ |  | 20 |  | 40 |  | 60 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Core 0 | T2 | T2 | T5 | T5 | T5 | T5 | T1 | T3 |
| Core 1 | T2 | T2 | T5 | T5 | T5 | T5 | T1 |  |
| Core 2 | T2 | T2 | T5 | T5 | T5 | T5 | T1 |  |
| Core 3 |  |  | T4 | T4 | T4 |  | T1 |  |

Figure 3: Schedule obtained by the CP/MISF algorithm.

| $t=0$ |  | 20 |  | 40 |  | 60 |  | 80 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Core 0 | T1 | T2 | T2 | T5 | T5 | T5 | T5 |  |
| Core 1 | T1 | T2 | T2 | T5 | T5 | T5 | T5 |  |
| Core 2 | T1 | T2 | T2 | T5 | T5 | T5 | T5 |  |
| Core 3 | T1 | T3 |  | T4 | T4 | T4 |  |  |

Figure 4: Schedule which takes into account the degree of data parallelism.

- P: The degree of data parallelism
- C: The length of critical path
- S: The number of immediate successors

Based on the three factors, the first algorithm proposed in this paper defines the priority of tasks as follows:

1. If task $i$ has a larger data parallelism than task $j$, task $i$ has a higher priority than task $j$.
2. In case tasks $i$ and $j$ has the same degree of data parallelism, if the critical path of task $i$ is longer than that of task $j$, task $i$ has a higher priority than task $j$.
3. In case tasks $i$ and $j$ has the same degree of parallelism and the same length of critical paths, if task $i$ has more immediate successors than task $j$, task $i$ has a higher priority than task $j$.

The algorithm based on the above priority is named $P C S$ since the three factors ( $\mathrm{P}, \mathrm{C}$ and S ) are prioritized in the order of P-C-S. Let PriorityPCS $S_{i}$ denote the priority of task $i$ in the PCS algorithm, where a higher value means a higher priority. A simple formula to define PriorityPCS ${ }_{i}$ is as follows.

$$
\begin{equation*}
\text { PriorityPCS }_{i}=U^{2} \cdot P_{i}+U \cdot C_{i}+S_{i} \tag{6}
\end{equation*}
$$

Here, $P_{i}, C_{i}$, and $S_{i}$ denote the values of $\mathrm{P}, \mathrm{C}$ and S factors for task $i$, and $U$ is a constant integer number which is larger than any of $P_{i}, C_{i}$, and $S_{i}$ for any $i$.

In the similar manner, we can define five algorithms $C P S, C S P, S C P, P S C$ and $S P C$ with different ordering of the three factors. The task priorities in the five algorithms are defined as follows:

$$
\begin{align*}
&{\text { Priority } C P S_{i}}=U^{2} \cdot C_{i}+U \cdot P_{i}+S_{i}  \tag{7}\\
&{\text { Priority } C S P_{i}}=U^{2} \cdot C_{i}+U \cdot S_{i}+P_{i}  \tag{8}\\
&{\text { Priority } S C P_{i}}=U^{2} \cdot S_{i}+U \cdot C_{i}+P_{i}  \tag{9}\\
& \text { PriorityPSC }_{i}=U^{2} \cdot P_{i}+U \cdot S_{i}+C_{i}  \tag{10}\\
& \text { PrioritySPC }_{i}=U^{2} \cdot S_{i}+U \cdot P_{i}+C_{i} \tag{11}
\end{align*}
$$

A common important feature in the six algorithms is that priorities are static. The priorities can be computed prior to scheduling, and they do not change during scheduling.


Figure 5: Averages of normalized schedule lengths for task graphs with 50 tasks.

The time complexity of the six algorithms is $O\left(N^{2}\right)$, where N is the number of tasks, assuming that the number of cores is constant. First, it takes $O\left(N^{2}\right)$ to compute the critical path lengths of the nodes. Then, we sort the nodes three times since we use three factors ( $\mathrm{P}, \mathrm{C}$ and S ), and each sorting takes $O(N \log N)$. Therefore, it takes $O\left(N^{2}\right)$ in order to compute the priorities of the nodes before running the list scheduling shown in Section 3.1. The list scheduling process is repeated $N$ times, and in each iteration, it takes $O(N)$ to update the ready list. Thus, we get the overall complexity of $O\left(N^{2}\right)$.

## 4 Experiments

We implemented the six algorithms in the C language, and tested their effectiveness. We used 43 task graphs from Standard Task Graph (STG) Set developed at Waseda University [6]. Forty out of the 43 task graphs are randomly generated ones, and the other three tasks are based on actual applications. Since tasks in STG do not assume data parallelism, we randomly assigned the degree of data parallelism to the tasks. The number of cores was changed from two to sixteen. In addition to the six algorithms presented in this paper, an integer linear programming (ILP) technique (see Section 2.2) was evaluated. In order to solve the ILP problems, IBM ILOG CPLEX 12.5 was used. Since exact solutions could not be found in a practical time, we limited the CPU time of CPLEX up to 60 minutes on dual Xeon processors (E5-2650, 2.00Hz, 128GB memory), and the best solution found at that time was compared with the six algorithms.

### 4.1 Results for Random Task Graphs

First, we conducted experimemts using 20 random task graphs, each of which consists of 50 tasks. Figure 5 shows the average schedule lengths of the 20 task graphs obtained by the six algorithms proposed in this paper. The schedule lengths are normalized to the PCS algorithm. This graph clearly shows the effectiveness of the PCS algorithm.

Table 1 shows detailed results for individual task graphs. The first column labeled as "Tid" shows the task ID, and the following columns show the lengths of the schedules obtained by the seven methods (the six algorithms proposed in this paper and the ILP method). For each benchmark, the best solution is shaded in yellow. $X$ in the ILP column means that no feasible solution was found within 60 minutes in CPU time. In many cases, the ILP method failed to find a feasible schedule


Figure 6: Averages of normalized schedule lengths for task graphs with 100 tasks.
within the limited time. Even when the ILP method found feasible schedules, they are lengthy. Although the PCS algorithm yields the best schedule results on average, Table 1 shows that the effectiveness of the six algorithms highly depends on the task graph.

Next, we conducted experimemts using 20 random task graphs, each of which consists of 100 tasks. Figure 6 shows the average schedule lengths of the 20 task graphs obtained by the six algorithms proposed in this paper. Again, this graph clearly shows the effectiveness of the PCS algorithm.

Table 2 shows detailed results for individual task graphs with 100 tasks. Compared with Table 1, the PCS algorithm yeilds best solutions in more cases, and the ILP method failed to find a feasible solution in more cases.

Table 1: Schedule lengths for task graphs with 50 tasks.

|  | 2 cores |  |  |  |  |  |  | 4 cores |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tid | PCS | CPS | CSP | SCP | PSC | SPC | ILP | PCS | CPS | CSP | SCP | PSC | SPC | ILP |
| 00 | 203 | 200 | 200 | 210 | 200 | 212 | 204 | 168 | 178 | 178 | 175 | 180 | 178 | X |
| 01 | 232 | 233 | 233 | 249 | 233 | 251 | 232 | 220 | 214 | 214 | 229 | 214 | 232 | X |
| 02 | 188 | 192 | 192 | 199 | 192 | 199 | 197 | 173 | 173 | 173 | 183 | 174 | 186 | 197 |
| 03 | 224 | 224 | 224 | 230 | 225 | 228 | 241 | 194 | 202 | 202 | 211 | 202 | 201 | X |
| 04 | 177 | 181 | 181 | 189 | 181 | 191 | 180 | 167 | 168 | 168 | 171 | 170 | 186 | X |
| 05 | 495 | 496 | 496 | 520 | 496 | 531 | 504 | 439 | 443 | 438 | 448 | 449 | 448 | 464 |
| 06 | 351 | 363 | 363 | 372 | 363 | 375 | 356 | 275 | 293 | 293 | 294 | 293 | 305 | X |
| 07 | 384 | 387 | 387 | 394 | 391 | 400 | 430 | 357 | 348 | 348 | 358 | 349 | 367 | X |
| 08 | 434 | 456 | 456 | 447 | 456 | 464 | 460 | 409 | 415 | 415 | 424 | 415 | 412 | 456 |
| 09 | 386 | 397 | 397 | 412 | 397 | 410 | 398 | 327 | 373 | 373 | 368 | 373 | 363 | X |
| 10 | 153 | 162 | 162 | 156 | 163 | 159 | 165 | 131 | 139 | 139 | 134 | 140 | 134 | X |
| 11 | 205 | 213 | 213 | 208 | 213 | 210 | 198 | 181 | 192 | 192 | 177 | 192 | 177 | 191 |
| 12 | 208 | 211 | 211 | 213 | 211 | 213 | 200 | 197 | 195 | 195 | 201 | 195 | 212 | X |
| 13 | 238 | 252 | 252 | 282 | 252 | 287 | 248 | 186 | 214 | 214 | 239 | 214 | 254 | X |
| 14 | 195 | 197 | 197 | 196 | 197 | 201 | 208 | 171 | 181 | 181 | 175 | 181 | 175 | X |
| 15 | 425 | 448 | 448 | 452 | 448 | 444 | 427 | 376 | 377 | 377 | 383 | 373 | 386 | 382 |
| 16 | 374 | 390 | 390 | 398 | 395 | 408 | 389 | 318 | 330 | 330 | 342 | 331 | 356 | 360 |
| 17 | 439 | 448 | 467 | 492 | 456 | 491 | 471 | 377 | 396 | 396 | 414 | 396 | 414 | X |
| 18 | 428 | 443 | 443 | 438 | 443 | 430 | 429 | 403 | 390 | 390 | 408 | 392 | 414 | 401 |
| 19 | 393 | 409 | 409 | 416 | 403 | 407 | 404 | 342 | 368 | 368 | 368 | 369 | 373 | X |
|  | 8 cores |  |  |  |  |  |  | 16 cores |  |  |  |  |  |  |
| Tid | PCS | CPS | CSP | SCP | PSC | SPC | ILP | PCS | CPS | CSP | SCP | PSC | SPC | ILP |
| 00 | 149 | 152 | 152 | 151 | 160 | 160 | X | 156 | 149 | 152 | 148 | 152 | 160 | 211 |
| 01 | 203 | 210 | 210 | 197 | 210 | 212 | X | 195 | 204 | 205 | 213 | 204 | 213 | 227 |
| 02 | 161 | 153 | 153 | 156 | 153 | 164 | X | 150 | 143 | 143 | 149 | 143 | 146 | 199 |
| 03 | 175 | 180 | 180 | 183 | 180 | 189 | X | 169 | 174 | 174 | 171 | 174 | 184 | 219 |
| 04 | 150 | 155 | 155 | 160 | 154 | 172 | X | 158 | 159 | 159 | 157 | 159 | 167 | 188 |
| 05 | 432 | 402 | 402 | 438 | 402 | 439 | X | 406 | 399 | 399 | 413 | 399 | 451 | 463 |
| 06 | 259 | 260 | 252 | 269 | 262 | 281 | X | 268 | 261 | 261 | 263 | 261 | 282 | 360 |
| 07 | 336 | 325 | 325 | 324 | 324 | 338 | X | 301 | 283 | 283 | 298 | 283 | 288 | 431 |
| 08 | 366 | 362 | 362 | 367 | 362 | 377 | X | 360 | 347 | 347 | 370 | 347 | 369 | 438 |
| 09 | 323 | 324 | 324 | 338 | 324 | 349 | X | 289 | 303 | 303 | 309 | 303 | 286 | 382 |
| 10 | 127 | 134 | 134 | 128 | 134 | 132 | 193 | 126 | 133 | 133 | 129 | 133 | 133 | 168 |
| 11 | 180 | 173 | 173 | 178 | 173 | 195 | X | 135 | 155 | 155 | 172 | 155 | 186 | 175 |
| 12 | 183 | 180 | 180 | 183 | 180 | 183 | X | 174 | 182 | 183 | 183 | 182 | 197 | 213 |
| 13 | 171 | 170 | 169 | 215 | 170 | 233 | X | 154 | 174 | 174 | 199 | 174 | 201 | 243 |
| 14 | 166 | 169 | 169 | 164 | 169 | 164 | X | 160 | 160 | 158 | 162 | 160 | 166 | 191 |
| 15 | 304 | 314 | 314 | 307 | 314 | 307 | X | 325 | 336 | 336 | 331 | 336 | 343 | 445 |
| 16 | 269 | 289 | 289 | 319 | 302 | 323 | X | 286 | 301 | 301 | 291 | 304 | 286 | 387 |
| 17 | 306 | 305 | 305 | 326 | 310 | 342 | X | 333 | 337 | 337 | 319 | 338 | 336 | 481 |
| 18 | 358 | 357 | 357 | 354 | 362 | 363 | 403 | 342 | 350 | 350 | 372 | 350 | 382 | 415 |
| 19 | 361 | 373 | 373 | 371 | 373 | 371 | X | 334 | 332 | 332 | 319 | 332 | 334 | 401 |

Table 2: Schedule lengths for task graphs with 100 tasks.

|  | 2 cores |  |  |  |  |  |  | 4 cores |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tid | PCS | CPS | CSP | SCP | PSC | SPC | ILP | PCS | CPS | CSP | SCP | PSC | SPC | ILP |
| 00 | 431 | 447 | 447 | 463 | 445 | 466 | X | 388 | 396 | 396 | 399 | 392 | 406 | X |
| 01 | 401 | 411 | 411 | 416 | 411 | 418 | X | 348 | 361 | 366 | 381 | 362 | 380 | X |
| 02 | 459 | 480 | 486 | 508 | 480 | 512 | X | 413 | 429 | 429 | 448 | 429 | 466 | X |
| 03 | 406 | 419 | 419 | 427 | 416 | 431 | 501 | 341 | 363 | 363 | 375 | 365 | 375 | X |
| 04 | 393 | 417 | 417 | 408 | 422 | 416 | 459 | 454 | 369 | 376 | 387 | 382 | 396 | X |
| 05 | 814 | 833 | 833 | 868 | 842 | 873 | X | 704 | 707 | 698 | 739 | 698 | 753 | X |
| 06 | 868 | 886 | 882 | 916 | 886 | 899 | 965 | 785 | 778 | 778 | 790 | 782 | 813 | X |
| 07 | 861 | 872 | 872 | 888 | 869 | 929 | 997 | 760 | 773 | 773 | 797 | 773 | 806 | X |
| 08 | 796 | 818 | 818 | 824 | 818 | 806 | X | 701 | 726 | 726 | 750 | 726 | 739 | X |
| 09 | 947 | 963 | 963 | 958 | 963 | 974 | X | 783 | 806 | 810 | 852 | 810 | 843 | X |
| 10 | 464 | 485 | 485 | 488 | 485 | 490 | 532 | 385 | 402 | 402 | 405 | 402 | 417 | X |
| 11 | 445 | 464 | 466 | 456 | 466 | 455 | X | 394 | 406 | 410 | 400 | 416 | 400 | X |
| 12 | 469 | 484 | 484 | 522 | 484 | 528 | 551 | 432 | 450 | 450 | 477 | 450 | 490 | X |
| 13 | 480 | 502 | 502 | 513 | 502 | 513 | X | 404 | 435 | 440 | 426 | 437 | 431 | X |
| 14 | 391 | 417 | 417 | 422 | 415 | 418 | X | 354 | 353 | 357 | 370 | 359 | 369 | X |
| 15 | 781 | 792 | 792 | 873 | 792 | 866 | X | 706 | 695 | 694 | 721 | 697 | 734 | X |
| 16 | 764 | 862 | 860 | 868 | 857 | 863 | X | 667 | 700 | 700 | 722 | 700 | 730 | X |
| 17 | 860 | 920 | 922 | 936 | 922 | 927 | X | 746 | 796 | 798 | 828 | 798 | 818 | X |
| 18 | 724 | 777 | 792 | 794 | 779 | 828 | X | 628 | 669 | 662 | 651 | 669 | 686 | X |
| 19 | 749 | 825 | 825 | 860 | 825 | 844 | 856 | 700 | 725 | 726 | 802 | 743 | 814 | X |
|  | 8 cores |  |  |  |  |  |  | 16 cores |  |  |  |  |  |  |
| Tid | PCS | CPS | CSP | SCP | PSC | SPC | ILP | PCS | CPS | CSP | SCP | PSC | SPC | ILP |
| 00 | 356 | 355 | 355 | 357 | 361 | 368 | X | 335 | 351 | 354 | 358 | 363 | 346 | 494 |
| 01 | 326 | 345 | 347 | 350 | 346 | 366 | X | 307 | 327 | 317 | 326 | 337 | 327 | 483 |
| 02 | 380 | 380 | 382 | 387 | 382 | 387 | X | 365 | 352 | 381 | 353 | 381 | 353 | 501 |
| 03 | 338 | 354 | 354 | 371 | 353 | 365 | X | 314 | 329 | 336 | 331 | 354 | 327 | 449 |
| 04 | 340 | 355 | 344 | 342 | 350 | 360 | X | 317 | 314 | 324 | 320 | 348 | 320 | 489 |
| 05 | 713 | 701 | 701 | 764 | 701 | 759 | X | 668 | 690 | 699 | 690 | 716 | 690 | 920 |
| 06 | 712 | 732 | 730 | 730 | 730 | 731 | X | 687 | 705 | 719 | 705 | 751 | 701 | 789 |
| 07 | 675 | 728 | 728 | 712 | 728 | 709 | X | 665 | 694 | 690 | 696 | 680 | 694 | 945 |
| 08 | 637 | 669 | 669 | 671 | 669 | 674 | X | 607 | 618 | 620 | 618 | 639 | 618 | 900 |
| 09 | 785 | 754 | 754 | 748 | 754 | 774 | X | 728 | 742 | 786 | 742 | 788 | 742 | 944 |
| 10 | 338 | 354 | 375 | 358 | 356 | 358 | X | 362 | 370 | 372 | 362 | 361 | 358 | 501 |
| 11 | 353 | 382 | 384 | 389 | 381 | 398 | X | 336 | 342 | 342 | 344 | 351 | 398 | 480 |
| 12 | 431 | 435 | 435 | 441 | 435 | 443 | X | 410 | 394 | 397 | 437 | 414 | 443 | 541 |
| 13 | 382 | 402 | 405 | 395 | 402 | 406 | X | 375 | 395 | 399 | 431 | 394 | 406 | 556 |
| 14 | 327 | 344 | 343 | 342 | 343 | 347 | X | 313 | 337 | 338 | 325 | 338 | 347 | 473 |
| 15 | 697 | 671 | 658 | 714 | 658 | 692 | X | 606 | 625 | 625 | 613 | 597 | 692 | 978 |
| 16 | 625 | 649 | 649 | 705 | 657 | 721 | X | 648 | 670 | 670 | 671 | 670 | 721 | 876 |
| 17 | 730 | 770 | 770 | 816 | 770 | 783 | X | 677 | 727 | 727 | 750 | 727 | 783 | 1024 |
| 18 | 657 | 668 | 668 | 673 | 668 | 679 | X | 591 | 644 | 652 | 615 | 652 | 679 | 832 |
| 19 | 679 | 705 | 701 | 801 | 701 | 775 | X | 676 | 682 | 686 | 731 | 690 | 775 | 796 |

Table 3: Schedule lengths for realistic task graphs.
(a) fpppp

|  | 2 cores | 4 cores | 8 cores | 16 cores |
| :---: | :---: | :---: | :---: | :---: |
| PCS | 5361 | 4881 | 4533 | 4487 |
| CPS | 5738 | 5152 | 4987 | 4905 |
| CSP | 5738 | 5152 | 4987 | 4905 |
| SCP | 5809 | 5108 | 4946 | 4899 |
| PSC | 5363 | 4884 | 4538 | 4531 |
| SPC | 5509 | 5032 | 4689 | 4623 |

(b) robot

|  | 2 cores | 4 cores | 8 cores | 16 cores |
| :---: | :---: | :---: | :---: | :---: |
| PCS | 1951 | 1739 | 1731 | 1615 |
| CPS | 1961 | 1769 | 1672 | 1641 |
| CSP | 1961 | 1769 | 1672 | 1641 |
| SCP | 1975 | 1791 | 1715 | 1637 |
| PSC | 1952 | 1767 | 1731 | 1615 |
| SPC | 2002 | 1783 | 1687 | 1627 |

(c) sparse

|  | 2 cores | 4 cores | 8 cores | 16 cores |
| :---: | :---: | :---: | :---: | :---: |
| PCS | 1458 | 1242 | 1132 | 1038 |
| CPS | 1442 | 1312 | 1222 | 1140 |
| CSP | 1442 | 1312 | 1222 | 1140 |
| SCP | 1454 | 1276 | 1172 | 1104 |
| PSC | 1458 | 1242 | 1136 | 1038 |
| SPC | 1454 | 1248 | 1166 | 1086 |

### 4.2 Results for Realistic Task Graphs

In addition to the random task graphs, we used three task graphs which are derived from realistic applications. The STG contains three task graphs based on realistic application programs, i.e., (a) a part of fpppp from in the SPEC benchmarks, (b) robot control and (c) sparse matrix solver [6]. The task graphs are generated by the OSCAR Parallelizing Compiler [7, 8, 9]. The task graphs of fpppp, robot and sparse contain 334 tasks, 88 tasks, and 96 tasks, respectively.

Table 3 shows the average schedule lengths for the three realistic task graphs. In order to understand more easily, we normalized the all results by the result of PCS, and converted the data to bar charts as Figures 7 (a), (b) and (c). We found the PCS algorithm yields good schedules in general. However, for robot on 8 cores and sparse on 2 cores, some others algorithms perform better than PCS.


Figure 7: Normalized schedule lengths for realistic task graphs.

## 5 Conclusions

This paper proposed six algorithms for scheduling tasks on multi/many-core processors. Unlike most of previous research efforts, the proposed algorithms schedule tasks which have data parallelism and run on multiple cores. The experimental results show that, among the six algorithms, the PCS algorithm yields the best schedule results on average.

In some task sets, the PCS algorithm does not yield good schedules. The effectiveness of the six algorithms heavily depends on the structure of task graphs. In the future, we will investigate the algorithms theoretically and compare them with optimal schedules in order to further improve the algorithms. Also, the current algorithms do not take into account communication costs, which should be addressed in the future.

## References

[1] E. G. Coffman: Computer and Job-shop Scheduling Theory, Wiley, 1976.
[2] H. Yang and S. Ha: ILP based data parallel multi-task mapping/scheduling technique for MPSoC, International SoC Design Conference (ISOCC), 2008.
[3] H. Yang and S. Ha: Pipelined data parallel task mapping/scheduling technique for MPSoC, Design Automation and Test in Europe (DATE), 2009.
[4] N. Vydyanathan, S. Krishnamoorthy, G. M. Sabin, U. V. Catalyurek, T. Kurc, P. Sadayappan, and J. H. Saltz: An integrated approach to locality-conscious processor allocation and scheduling of mixed-parallel applications, IEEE Trans. on Parallel and Distributed Systems, vol. 20, no. 8, pp. 1158-1172, Aug. 2009.
[5] H. Kasahara and S. Narita: Practical multiprocessor scheduling algorithms for efficient parallel processing, IEEE Trans. on Computers, vol. C-33, no. 11, Nov. 1984.
[6] http://www.kasahara.elec.waseda.ac.jp/schedule/ (Last accessed: July 24, 2013)
[7] H. Kasahara, H. Honda and S. Narita: Parallel Processing of Near Fine Grain Tasks Using Static Scheduling on OSCAR, Proc. IEEE ACM Supercomputing '90, 1990.
[8] H. Kasahra, H. Honda, A. Mogi, A. Ogura, K. Fujiwara and S. Narita: A Multi-grain Parallelizing Compilation Scheme for OSCAR, Proc. 4th Workshop on Languages and Compilers for Parallel Computing, 1991.
[9] A. Yoshida, K. Koshizuka and H. Kasahara: Data-Localization for Fortran Macrodataflow Computation Using Partial Static Task Assignment, Proc. 10th ACM Int'l Conf. on Supercomputing, pp. 61-68, 1996.

